C3.3 PDE and Systems of ODE
Total Marks: 100, Theory: 60, IA: 20, Practical: 20
Credit: 4+2=6;
(L=4, P=4, T=0)

Objectives: After going through this course the students will be able to

- make mathematical formulations and their solutions of various physical problems;
- design mathematical models used in heat, wave.
- Describe the Laplace equation and their solutions.

Unit-1

Marks: 25, Contact hrs: 25

Partial Differential Equations – Basic concepts and Definitions, Mathematical Problems. First- Order Equations: Classification, Construction and Geometrical Interpretation. Method of Characteristics for obtaining General Solution of Quasi Linear Equations. Non-linear partial differential equations, Charpit's method & Jacobi's method Canonical Forms of First-order Linear Equations. Method of Separation of Variables for solving first order partial differential equations.

Unit-2

Marks: 12, Contact hrs: 10

Classifications of second order linear equations as hyperbolic, parabolic or elliptic. Derivations of Heat equation, Wave equation and Laplace equation and their solutions Reduction of second order Linear Equations to canonical forms.

Unit-3

Marks: 8, Contact hrs: 10

Method of separation of variables, Solving the Vibrating String Problem, Solving the Heat Conduction problem

Unit-4

Marks: 15, Contact hrs: 15

Systems of linear differential equations, types of linear systems, differential operators, an operator method for linear systems with constant coefficients, Basic Theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two Equations in two unknown functions.The method of successive approximations.

List of Practicals (using any software) **Marks: 20** Contact hrs. 30

(i) Solution of Cauchy problem for first order PDE.
(ii) Finding the characteristics for the first order PDE
(iii) Plot the integral surfaces of a given first order PDE with initial data.

(iv) Solution of wave equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ for the following associate conditions

(a) $u(x,0) = \varphi(x)$, $u_t(x,0) = \psi(x)$, $x \in R, t \to 0$
(b) $u(x,0) = \varphi(x)$, $u_t(x,0) = \psi(x)$, $u(0,t) = 0$, $x \in (0,\infty), t > 0$
(c) $u(x,0) = \phi(x)$, $u_t(x,0) = \psi(x)$, $u(x,0) = 0$, $x \in (0,\infty), t > 0$;
(d) $u(x,0) = \phi(x)$, $u_t(x,0) = \psi(x)$, $u(0,t) = 0$, $u(1,t) = 0$, $0 < x < l, l > 0$

(v) Solution of wave equation $\frac{\partial u}{\partial t} - k^2 \frac{\partial^2 u}{\partial x^2} = 0$ for the following associate conditions

(a) $u(x,0) = \varphi(x)$, $u(0,t) = a$, $u(l,t) = b$, $0 < x < l, t > 0$
(b) $u(x,0) = \varphi(x)$, $x \in R, T > t > 0$
(c) $u(x,0) = \varphi(x)$, $u(0,t) = a$, $x \in (0,\infty), t \geq 0$;

Text Books:

1. S.L. Ross, *Differential equations,* 3rd Ed., John Wiley and Sons, India,2004.
2. I. N. Sneddon, Elements of Partial Differential Equations, Dover Publications, 2006.

Reference Books:

1. T. Myint-U and L. Debnath, *Linear Partial Differential Equations for Scientists and Engineers,* 4th edition, Springer, Indian reprint,2006.
2. M. L Abell, J. P Braselton, *Differential equations with MATHEMATICA,* 3rd Ed., Elsevier Academic Press,2004.

Guideline:

Unit 1 [2] Chapter 2
Unit 2 [1] Chapter 14.1,14.3
Unit 4 [1] Chapter 7.1 –7.4; 8.3, 8.4